Investigating the suitability of the negative log-likelihood term for the catch-at-length data in the hake assessment model

A. Ross-Gillespie and D.S. Butterworth

Summary

Various technical improvements are proposed to the way catch-at-length (CAL) data are treated in fitting the hake assessment model in preparation for finalising the Operating Models for the hake OMP revision. This work has been conducted in collaboration with OLRAC and their code-checking exercise, with near identical results achieved. Results suggest the *M. paradoxus* resource to be robustly estimated to be at least 10% above B_{MSY} at present for the Reference Case; similar estimates for *M. capensis* are also above B_{MSY}, though more variable in sensitivity tests.

Background

During the course of the OLRAC-MARAM code checking exercise conducted in 2017, the suitability of the manner in which the catch-at-length (CAL) proportions data had been incorporated into the MARAM model came under review. Previously, code had been written so that if the observed proportion for any given length group was zero, that length group was grouped with a neighbouring length group, with the corresponding model-predicted proportion receiving the same treatment. As most of the observed zeros are in the tails of the CAL distributions, this procedure in effect created year-varying plus and minus groups, which is *inter alia* not that desirable from a coding viewpoint. Subsequently, the approach was modified so that all the length groups (and thus all the zeros in the observed CAL dataset) were included in the negative log-likelihood calculations. This approach resulted in many near-zero cells for residuals and consequently negatively biased CAL sigma values, which in turn resulted in very large negative log-likelihood contributions for the CAL data (the sigma values were compared to those that one would get if one were to simply exclude (as opposed to group) all the length cells for which the observed CAL proportion is zero; the latter (referenced as the “exclude zeros” approach) being considered a scenario roughly unbiased by zeros).

New plus-minus groups

To address the issue of the present negatively biased sigmas, we are proposing to impose new plus-minus groups that are data-type dependent, but year-independent. These plus and minus groups were selected (a)based on plots of the observed and model predicted (from the Rademeyer RC results) CAL proportions (see Figure 1) and (b)so that the sigmas from a model run implementing the plus-minus groups were of similar size to those of the original “exclude zeros” approach. The proposed plus and minus groups are listed in Table 1, and the assessment model was re-run with these new plus-minus groups (**Run 3** of Table 2).

Homoscedasticity of the residuals

In addition to the above investigation of how best to define the plus-minus groups, it was found that the residuals of the fits to the CAL data appeared to be somewhat heteroscedastic (see Figure 2). This raised the question of whether the square root transformation of the observed and model-predicted CAL proportions in the negative log-likelihood is the most appropriate.

To seek a transformation that achieved CAL residual homoscedasticity as best as possible, the residuals for each series were first detrended by subtraction of an 11-point running mean, following which the variance of the detrended residuals was evaluated in a "running-mean-type-fashion", i.e. the detrended residuals were plotted against the model-predicted CAL proportions (pmod) and for each point p_i in the sorted pmod vector, a measure of variance was calculated for p_i by computing the variance for a subset of the detrended residuals consisting of the residual for p_i and those corresponding to the five points immediately to the left and right of p_i. Then the power parameter X (where residuals=Obs^X-Pred^X) was varied and the final power value selected was determined as that for which the variance of the detrended residuals was as near constant as possible (see Figure 2); this was identified by the minimum CV across a range of values of X . A histogram was constructed with the 'optimum' powers obtained for each series of CAL data (Figure 3), which yielded upper and lower quartiles at powers of roughly 0.2 and 0.5 respectively. The hake assessment model was then re-run with the plus-minus groups from Table 1together with CAL powers of 0.5, 0.35 and 0.2 (**Runs 3, 4 and 5** of Table 2).

Finally as the 0.1 multiplier for the -lnL contributions from the CAL data is somewhat arbitrary, we checked sensitivity of results to that (**Runs 6 and 7** of Table 2). One last run was conducted whereby the new plus-minus groups were used with a power of 0.35 in the CAL negative log-likelihood equation and additionally the natural mortality vector was replaced by the 1984-2014 average natural mortality from the Ross-Gillespie (2016) hake predation model (**Run 8** of Table 2). This last run is purely indicative at this stage, not final, but unlikely to be too different from what will result once the hake predation model and hence the M-at-age vectors are finalised.

Table 2 lists key statistics of these different runs, while Figure 4 shows biomass trajectories for a selection of the runs.

One issue that still warrants further investigation is possible multimodality of the model likelihood. Runs 4a and 4b have virtually the same negative log-likelihood, but the *M. capensis* depletion estimates are notably different. A high priority for further work is implementing the Baranov formulation for the catch equation in this model, as it is hoped that this might help with convergence issues through use of Punt's solution procedure which renders the problem less "stiff". We hope that this might both ease achievement of convergence and render the results more reliable when we come to finalise the Operating Model fits for the 2018 hake OMP review.

In summary, approval is needed from the DWG on the following aspects in order to move forward with the hake OMP review:

1. The plus/minus group specifications (the proposed plus/minus groups are given in **Table 1**)
2. The CAL likelihood specification (the proposal is to use a power of 0.35)
3. How to deal with the multimodality in the context of the OMs for the OMP revision, and the reporting results to MSC
4. The planned change from the Pope to the Baranov formulation of the catch equation.

Implications for the status of the two hake populations

The OLRAC code checking exercise brought to light that the Reference Case run had not fully converged. With further minimisation, a better minimum was found, which corresponded to slightly better current status of the *M. paradoxus* resource with respect to both carrying capacity K and B_{MSY} in terms of the mature female component of the population.

This improved estimate of current status for *M. paradoxus* of at least 10% above B_{MSY} remains robustly determined across a wide-ish range of sensitivity tests reported in Table 2. *M capensis* also remains robustly estimated to exceed B_{MSY}, though those estimates are more variable across these sensitivities.

Table 1: Proposed new plus and minus groups for the different data types. CAL data for length bins less than or equal to the minus group and larger than or equal to the plus group are grouped together. Lengths are given in cm.

1. Commercial sex-aggregated	Minus group	Plus group
West Coast offshore	19	81
South Coast offshore	19	81
South Coast inshore	19	81
West Coast longline	37	101
South Coast Longline	37	101
2. Commercial sex-disaggregated	**Minus group**	**Plus group**
WC longline M. paradoxus	37	101
WC longline M. capensis	37	101
SC longline M. paradoxus	37	101
SC longline M. capensis	37	101
3. Survey sex-aggregated	**Minus group**	**Plus group**
WC summer M. paradoxus	5	75
WC winter M. paradoxus	5	75
SC spring M. paradoxus	11	75
SC autumn M. paradoxus	11	75
WC summer M. capensis	5	75
WC winter M. capensis	5	75
SC spring M. capensis	5	75
SC autumn M. capensis	5	75
4. Survey sex-disaggregated	**Minus group**	**Plus group**
WC summer M. paradoxus	5	75
SC spring M. paradoxus	11	75
SC autumn M. paradoxus	11	75
WC summer M. capensis	5	75
SC spring M. capensis	5	75
SC autumn M. capensis	5	75

Table 2: Summary statistics for the different runs. Numbers 1, 2, 2b and 2c are all based on the same code and correspond to the original plus-minus groups and CAL weighting. Number 3 introduces the proposed new plus-minus groups. Number 4 is the proposed new Reference Case where a power of 0.35 is used for the catch-at-length terms in the negative log-likelihood, instead of the conventional square root (0.5 power). Number 4 and 4b have identical code, but different MLEs resulting from jittering. Numbers 5-8 are explained in the Run column.

		Negative log-likelihood			*M. paradoxus*					*M. capensis*				
	Run	Total -lnL	CAL -lnL	Comp. -lnL	Bsp_{2017}	K^{sp}	B_{MSY}	$\mathbf{B^{sp}_{2017}/K^{sp}}$	$\mathbf{B^{sp}_{2017}/B_{MSY}}$	Bsp_{2017}	K^{sp}	B_{MSY}	$\mathbf{B^{sp}_{2017}/K^{sp}}$	$\mathbf{B^{sp}_{2017}/B_{MSY}}$
1	Rebecca RC	-5244.1	-5110.4	-133.7	112	547	109	**0.20**	**1.03**	120	187	33	**0.64**	**3.62**
2	Rebecca's model, best MLE found to date	-5251.5	-5119.1	-132.4	127	515	115	**0.25**	**1.11**	141	196	63	**0.72**	**2.23**
2b	OLRAC MLE	-5248.4	-5119.4	-128.9	131	518	117	**0.25**	**1.11**	140	194	62	**0.72**	**2.24**
2c	Another jittered MLE	-5250.8	-5119.1	-131.7	121	532	103	**0.23**	**1.17**	120	187	33	**0.64**	**3.64**
3	Introducing plus-minus groups, CAL power 0.5	-3541.6	-3396.5	-145.1	132	511	118	**0.26**	**1.12**	117	187	32	**0.63**	**3.68**
4a	Plus-minus groups, CAL power 0.35 run A	-3172.6	-3029.6	-143	129	530	109	**0.24**	**1.18**	142	201	63	**0.71**	**2.26**
4b	Plus-minus groups, CAL power 0.35 run B	-3172.8	-3028.5	-144.4	132	512	112	**0.26**	**1.17**	117	186	32	**0.63**	**3.65**
5	Plus-minus groups, CAL power 0.20	-2814.0	-2667.5	-146.5	135	513	111	**0.26**	**1.21**	116	184	31	**0.63**	**3.79**
6	No. 4 with CAL weighting of 0.2 instead of 0.1	-6204.9	-6078.1	-126.8	125	503	110	**0.25**	**1.14**	166	228	54	**0.73**	**3.06**
7	No. 4 with CAL weighting of 0.05 instead of 0.1	-1662.0	-1505.0	-157.0	137	520	111	**0.26**	**1.24**	110	183	29	**0.60**	**3.79**
8	No. 4 run with predation mortality rates	-3157.1	-3017.3	-139.8	118	539	88	**0.22**	**1.34**	163	231	73	**0.70**	**2.24**

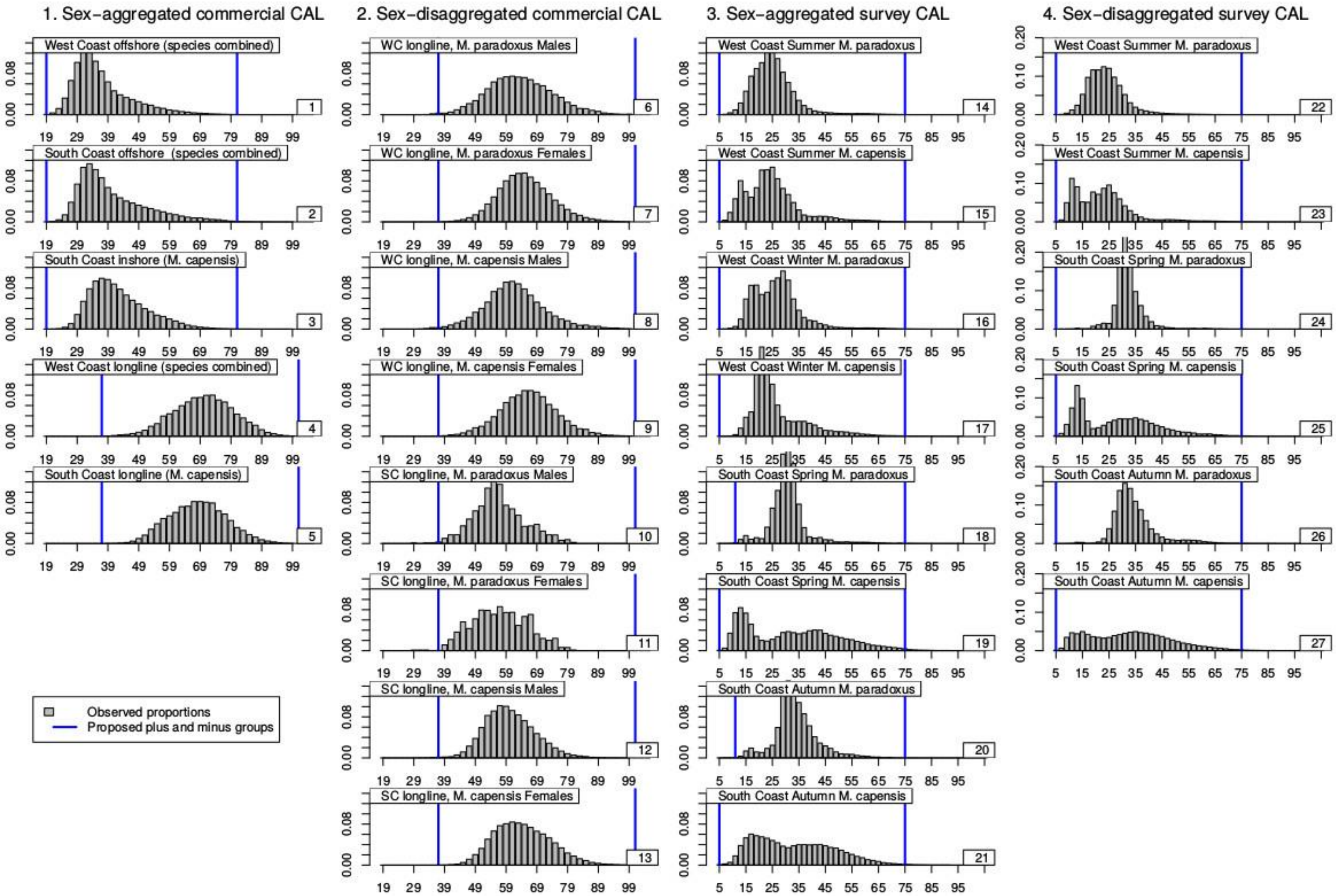

Figure 1: Histograms of the year-averaged observed catch-at-length proportions, with the proposed plus-minus groups indicated by the vertical blue lines.

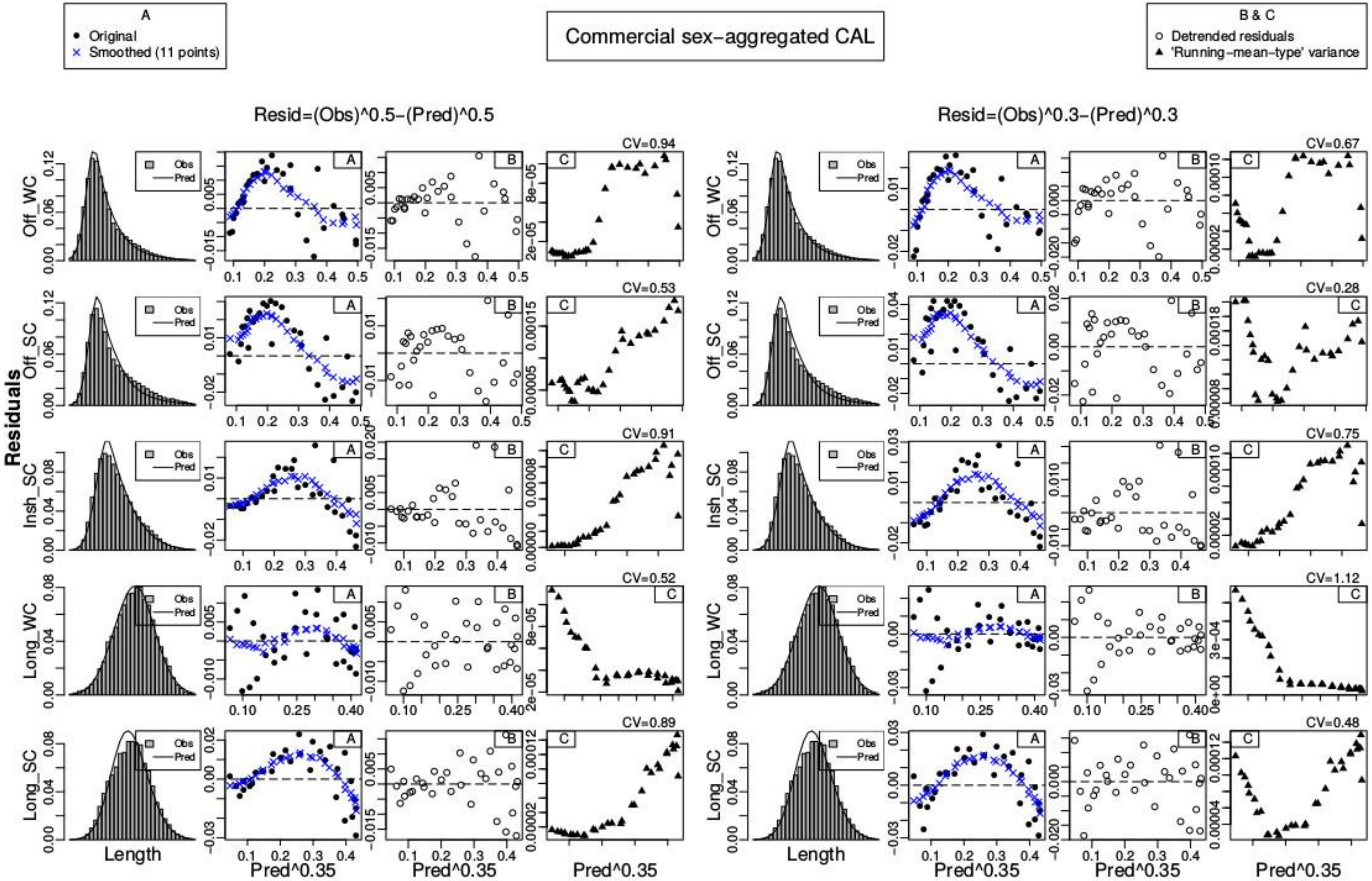

Figure 2: An illustration of the procedure followed to obtain an optimal power for the CAL negative log-likelihood term. The model-predicted values are taken from the fits of the Rademeyer RC model, and results are shown for two different values of X (0.5 and 0.35) where Residuals=Obs^X – Pred^X. The first column shows the data and model fits. The second column (A) shows the residuals with solid circles and the smoothed residuals (smoothing achieved by use of an 11-point running mean) are shown by blue crosses. The third column (B) shows the detrended residuals (residuals less smoothed residuals). The fourth column (C) shows the 'running-mean-type' variance of the detrended residuals in column (B).

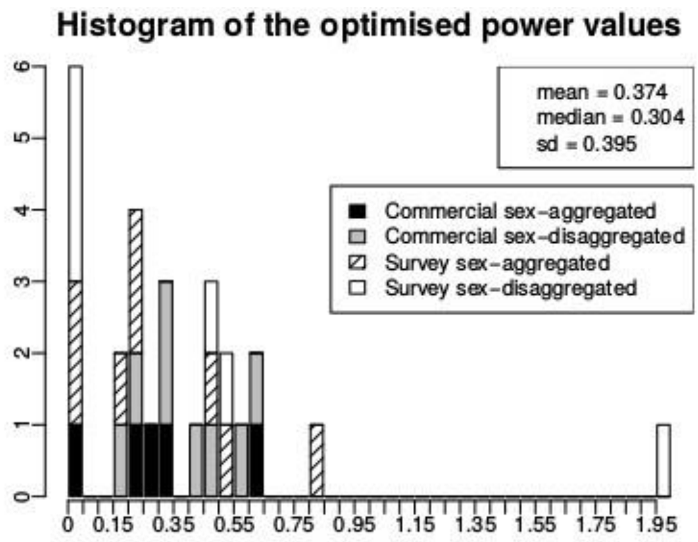

Figure 3: Histogram showing the optimised power value for each data type. The optimised power value was found by searching for the value of the power X (where Residuals=Obs^X-Pred^X) so that the CV of the "running-mean-type" variance (column C of Figure 2) was minimised.

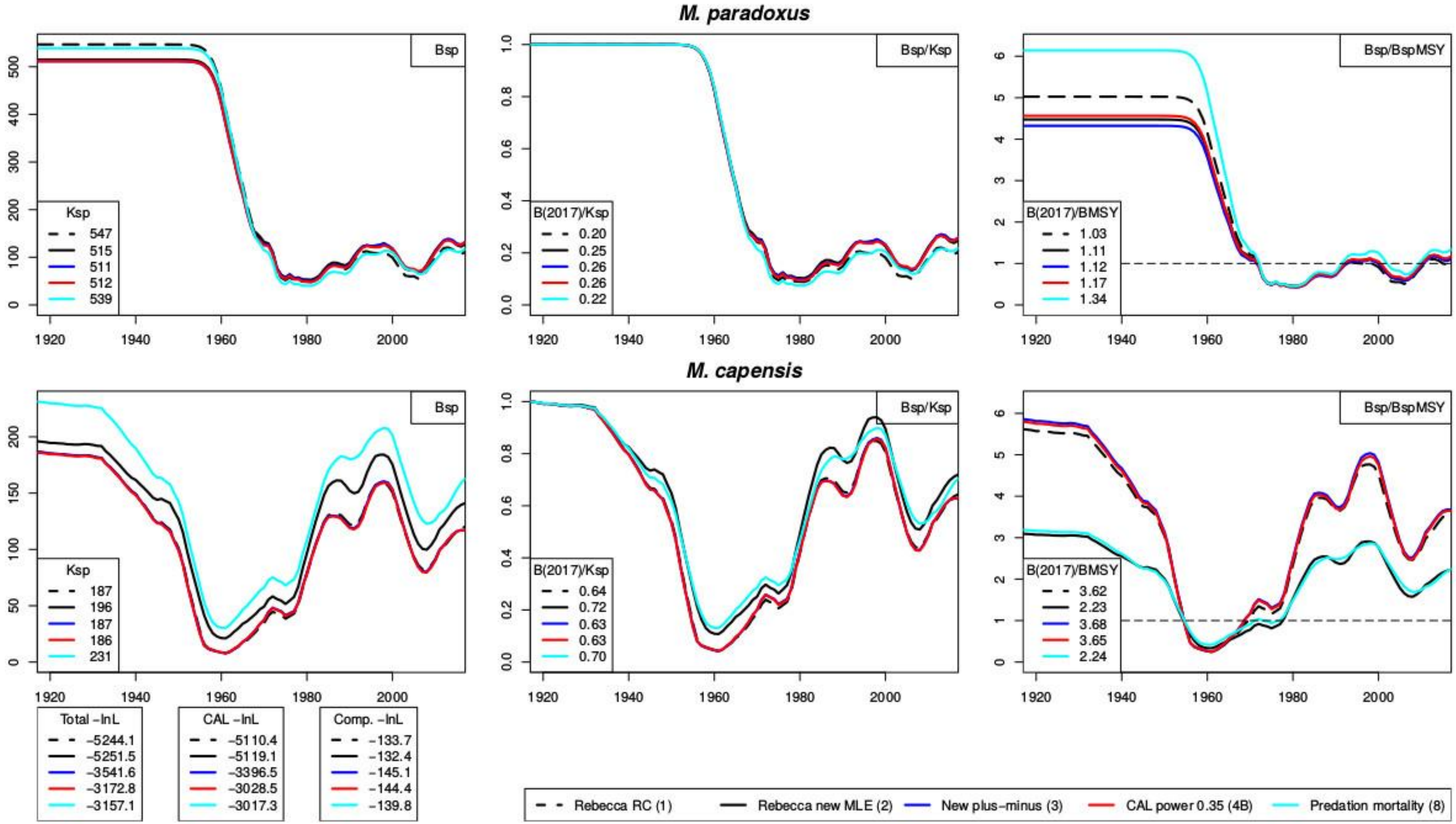

Figure 4: Spawning biomass trajectories for a selection of runs.

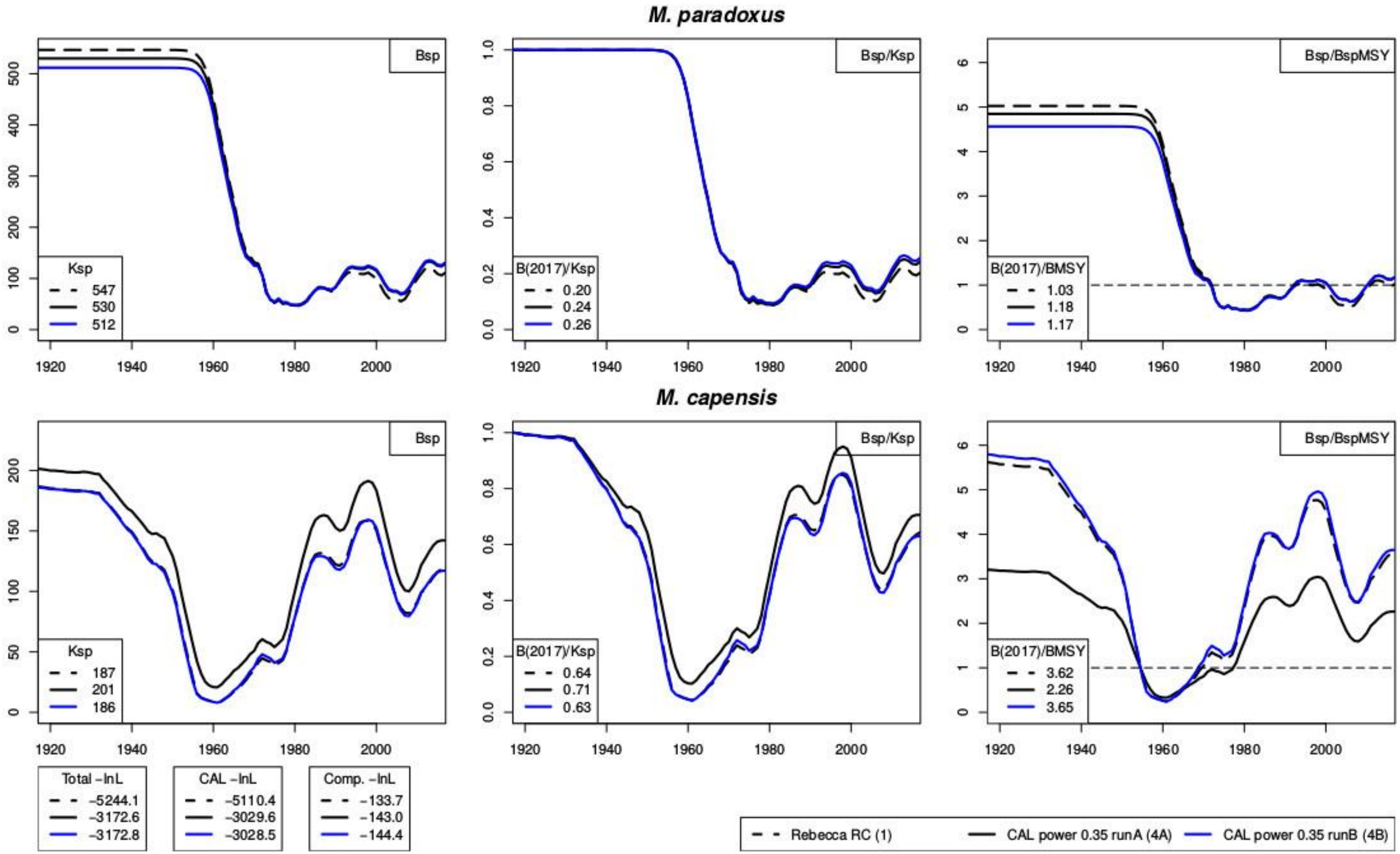

Figure 5: Spawning biomass trajectory for runs (1), (4a) and (4b). The difference between the negative log-likelihood between runs (4a) and (4b) is less than 1 point, suggesting the likelihood surface may be multimodal.